Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Friday 19 January 2024

Afternoon (Time: 1 hour 30 minutes) | **Paper reference** **WME03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$, and give your answer to either two significant figures or three significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1. A spacecraft S of mass m moves in a straight line towards the centre, O, of a planet.

The planet is modelled as a fixed sphere of radius R.

The spacecraft S is modelled as a particle.
The gravitational force of the planet is the only force acting on S.

When S is a distance x $(x \geqslant R)$ from O

- the gravitational force is directed towards O and has magnitude $\dfrac{mgR^2}{2x^2}$
- the speed of S is v

(a) Show that

$$v^2 = \frac{gR^2}{x} + C$$

where C is a constant.

(3)

When $x = 3R$, $v = \sqrt{3gR}$

(b) Find, in terms of g and R, the speed of S as it hits the surface of the planet.

(3)

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Question 1 continued

(Total for Question 1 is 6 marks)

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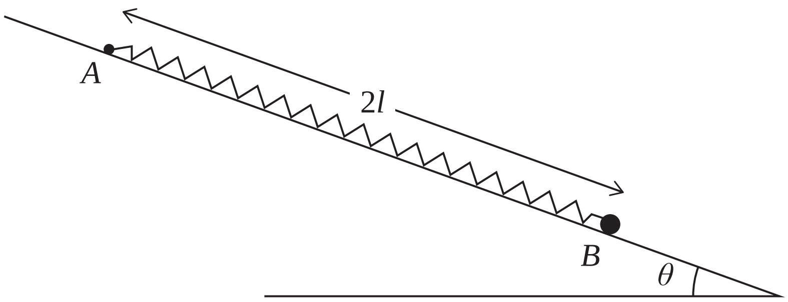

Figure 1

A light elastic **spring** has natural length l and modulus of elasticity λ
One end of the spring is attached to a point A on a smooth plane.

The plane is inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

A particle P of mass m is attached to the other end of the spring.

Initially P is held at the point B on the plane, where AB is a line of greatest slope of the plane.

The point B is lower than A and $AB = 2l$, as shown in Figure 1.

The particle is released from rest at B and first comes to instantaneous rest at the point C on AB, where $AC = 0.7l$

(a) Use the principle of conservation of mechanical energy to show that

$$\lambda = \frac{100}{91}mg$$

(5)

(b) Find the acceleration of P when it is released from rest at B.

(4)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

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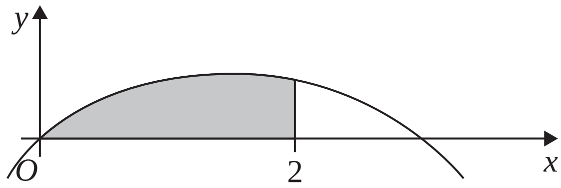

Figure 2

The shaded region in Figure 2 is bounded by the x-axis, the line with equation $x = 2$ and the curve with equation $y = \frac{1}{4}x(3 - x)$.

This region is rotated through 2π radians about the x-axis, to form a solid of revolution which is used to model a uniform solid S.

The volume of S is $\frac{2}{5}\pi$

(a) Use the model and algebraic integration to show that the x coordinate of the centre of mass of S is $\frac{31}{24}$

(5)

The solid S is placed with its circular face on a rough plane which is inclined at $\alpha°$ to the horizontal. The plane is sufficiently rough to prevent S from sliding.

The solid S is on the point of toppling.

(b) Find the value of α

(3)

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Question 3 continued

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Question 3 continued

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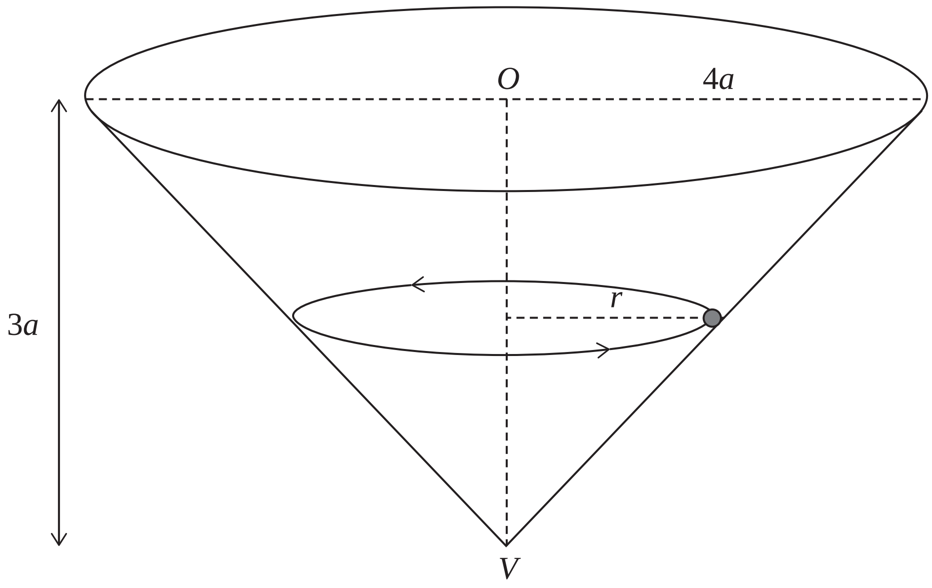

Figure 3

Figure 3 shows a thin hollow right circular cone fixed with its circular rim horizontal.

The centre of the circular rim is O. The vertex V of the cone is vertically below O.

The radius of the circular rim is $4a$ and $OV = 3a$.

A particle P of mass m moves in a horizontal circle of radius r $(0 < r < 4a)$ on the inner surface of the cone.

The coefficient of friction between P and the inner surface of the cone is $\frac{1}{4}$

The particle moves with a constant angular speed.

Show that the maximum possible angular speed is $\sqrt{\frac{16g}{13r}}$

(9)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 9 marks)

5. (a) Use algebraic integration to show that the centre of mass of a uniform semicircular disc of radius r and centre O is at a distance $\dfrac{4r}{3\pi}$ from the diameter through O

[You may assume, without proof, that the area of a circle of radius r is πr^2]

(5)

A uniform lamina L is in the shape of a semicircle with centre B and diameter $AC = 8a$. The semicircle with diameter AB is removed from L and attached to the straight edge BC to form the template T, shown shaded in Figure 4.

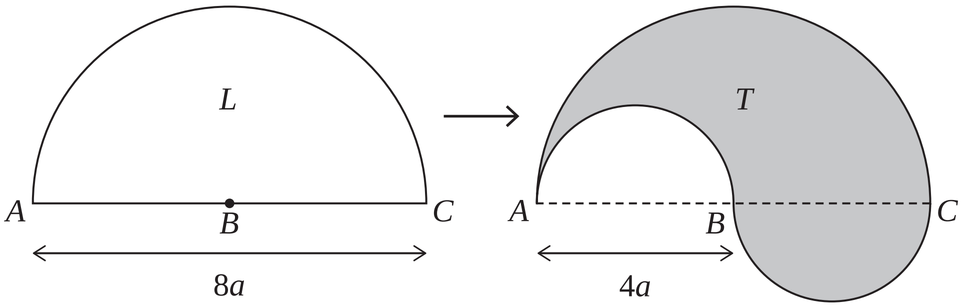

Figure 4

The distance of the centre of mass of T from AC is d.

(b) Show that $d = \dfrac{4a}{\pi}$

(5)

The template T is freely suspended from A and hangs in equilibrium with AC at an angle θ to the downward vertical.

(c) Find the exact value of $\tan\theta$

(6)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 16 marks)

6. The fixed point A is **vertically above** the fixed point B, with $AB = 3l$

A light elastic string has natural length l and modulus of elasticity $4mg$
One end of the string is attached to A and the other end is attached to a particle P of mass m

A second light elastic string also has natural length l and modulus of elasticity $4mg$
One end of this string is attached to P and the other end is attached to B.

Initially P rests in equilibrium at the point E, where AEB is a **vertical** straight line.

(a) Show that $AE = \frac{13}{8}l$

(4)

The particle P is now held at the point that is a distance $2l$ vertically below A and released from rest.

At time t, the vertical displacement of P from E is x, where x is measured vertically downwards.

(b) Show that $\ddot{x} = -\frac{8g}{l}x$

(4)

(c) Find, in terms of g and l, the speed of P when it is $\frac{1}{8}l$ below E.

(3)

(d) Find the length of time, in each complete oscillation, for which P is more than $1.5l$ from A, giving your answer in terms of g and l

(3)

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Question 6 continued

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Question 6 continued

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7.

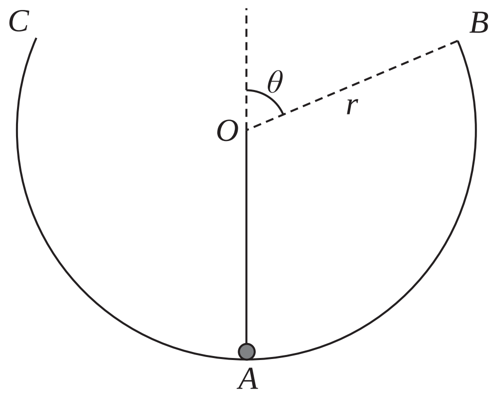

Figure 5

A thin smooth hollow spherical shell has centre O and radius r. Part of the shell is removed to form a bowl with a plane circular rim. The bowl is fixed with the circular rim uppermost and horizontal. The point A is the lowest point of the bowl, as shown in Figure 5.

The point B is on the rim of the bowl, with OB at an angle θ to the upward vertical, where $\tan\theta = \frac{12}{5}$

A small ball is placed in the bowl at A. The ball is projected from A with horizontal speed u and moves in the vertical plane AOB. The ball stays in contact with the bowl until it reaches B.

At the instant when the ball reaches B, the speed of the ball is v.

By modelling the ball as a particle and ignoring air resistance,

(a) use the principle of conservation of mechanical energy to show that

$$v^2 = u^2 - \frac{36}{13}gr$$

(3)

(b) show that $u^2 \geqslant \frac{41}{13}gr$

(4)

The point C is such that BC is a diameter of the rim of the bowl.

Given that $u^2 = 4gr$

(c) use the model to show that, after leaving the inner surface of the bowl at B, the ball falls back into the bowl before reaching C.

(6)

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

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